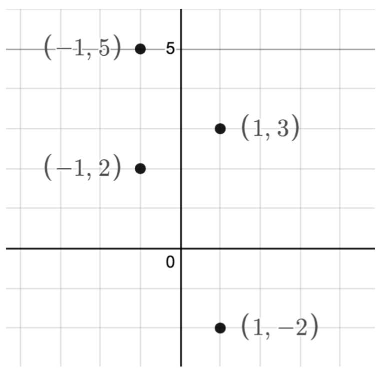
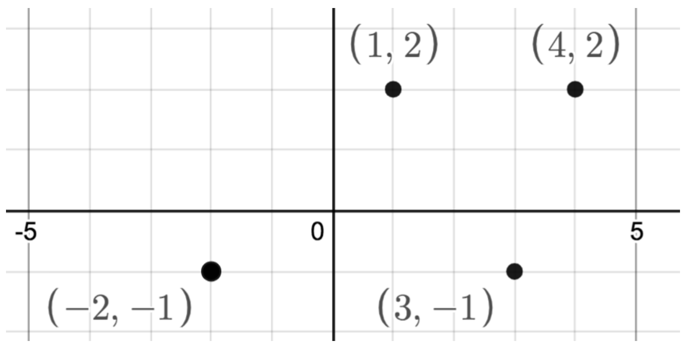
5.1 Area Between Curves (development in book differs)

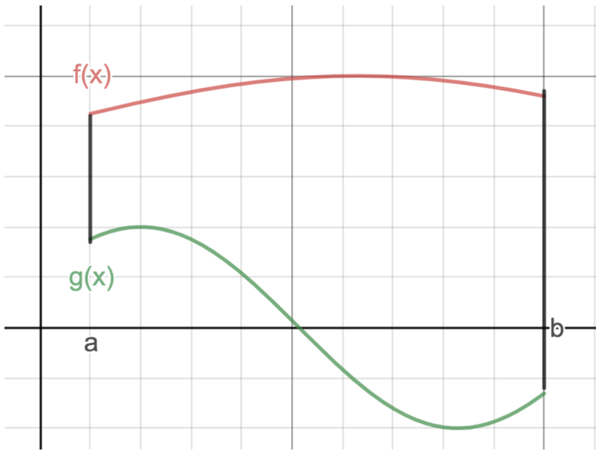
Finding vertical distance:



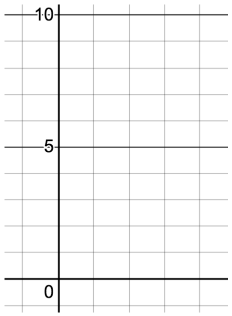
Finding Horizontal distance:



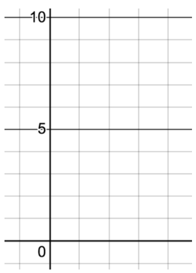
Suppose $f(x)$ and $g(x)$ are continuous on [a, b] with $f(x) \geq g(x)$ for all x in [a, b]. Find the area enclosed between these curves over [a, b].



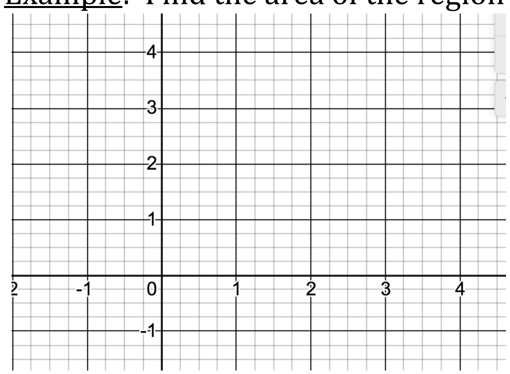
Example: Find the area of the region bounded by $y=x+6$ and $y=x^2$ over [0,2] **(Ans 34/3)**



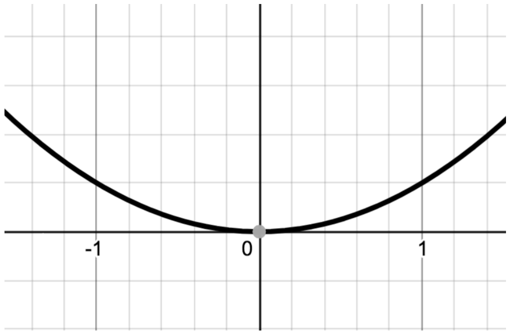
Example: Find the area of the region bounded by $y=x$ and $y=x^2$. **(Ans 1/6)**

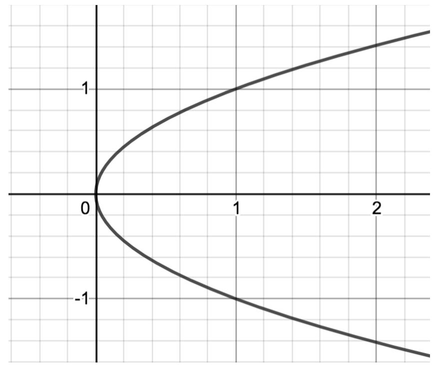


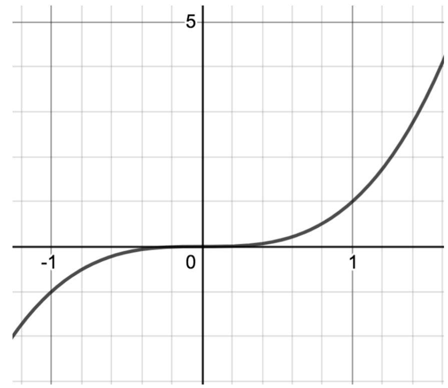
Example: Find the area of the region bounded by $x+y=4$, $y=2$ and $y=x^2+2$ **(Ans 5/6)**



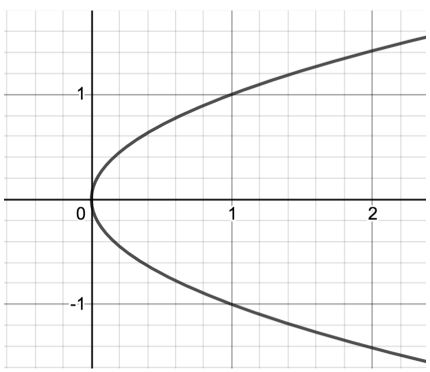
Consider looking at functions from a different point of view:





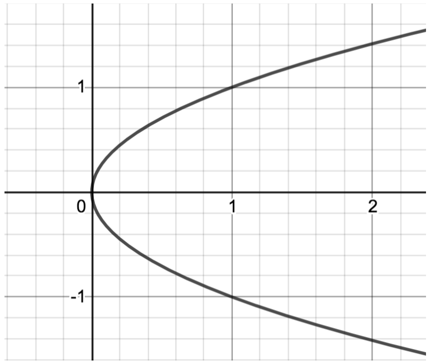


Some curves make more sense to view with y being the independent variable and x being a function of y.

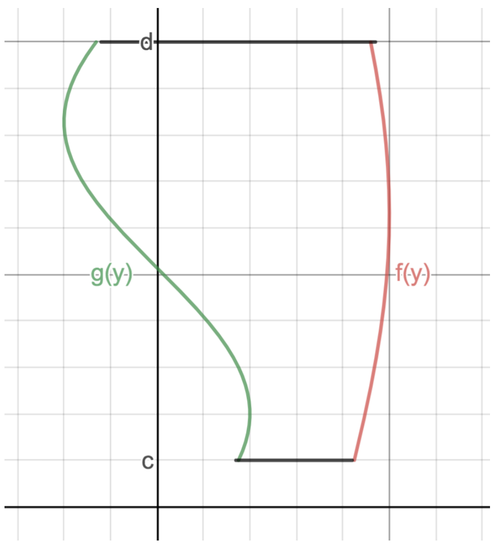


x	y

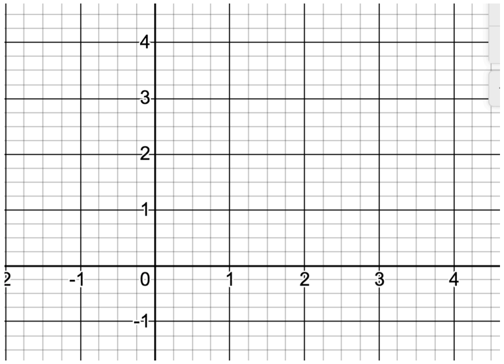
Example: Find the area of the region bounded by $y^2 = x$ and the y axis for $-1 \le y \le 1$ **(Ans 2/3)**



In general suppose $f(y)$ and g(y) are continuous on $c \le y \le d$ with $f(y) \le g(y)$ for all y in [c, d]. Find the area enclosed between these curves over [c, d].

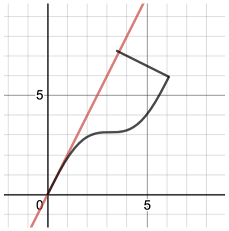


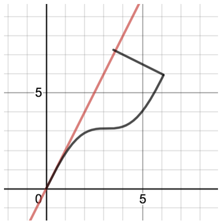
Redo example, this time with respect to y: Find the area of the region bounded by $x+y=4$, $y=2$ and $y=x^2+2$



5.2: Volume by Slicing and The Disk Method

Solid of Revolution
When a plane region is revolved around a line, called the axis of revolution, the resulting solid is called a solid of revolution.



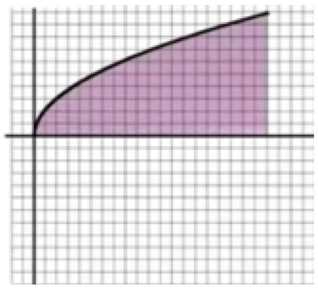


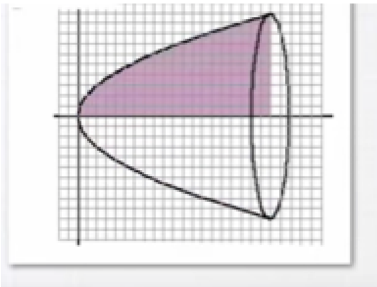
We will be computing volume for solids of revolutions whose axes are horizontal or vertical. See "Example Solids of Revolutions" Canvas. http://www.cecm.sfu.ca/~nbruin/math152/SOR/solids-of-rev.html

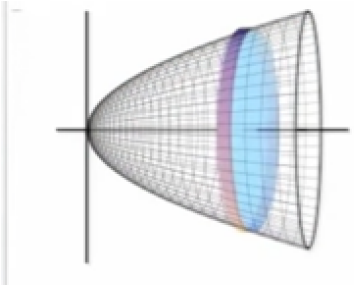
---

Disk Method, revolve about X AXIS

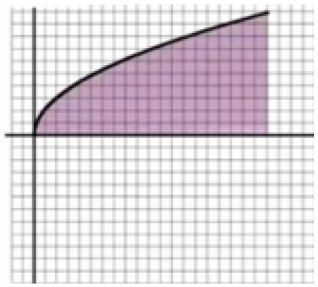
Consider a $f(x) \geq 0$, continuous on [a,b], revolve about the x axis.





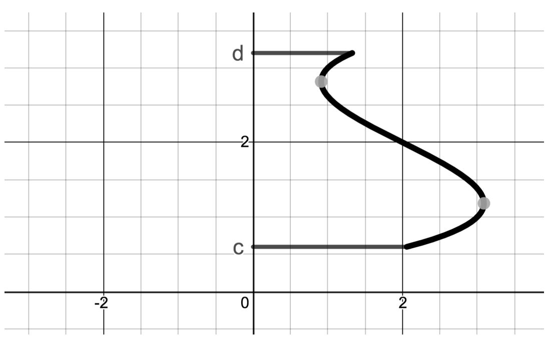


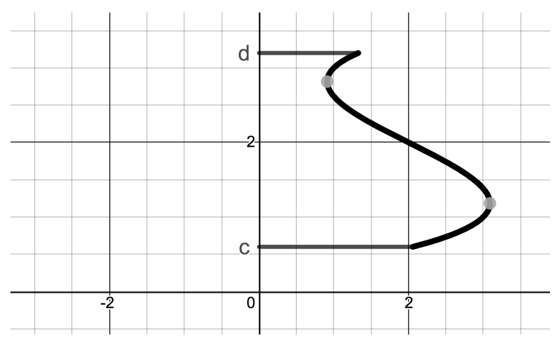
See disk method video on 5A page (first 40 seconds). Consider revolution of typical rectangle.
Example: The region bound by The picture can't be displayed., x=4 and x axis is revolved about the x axis. Find the volume:



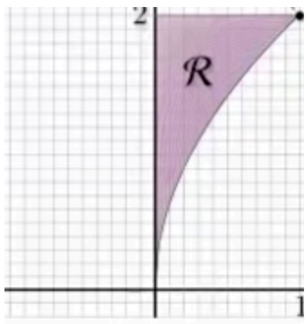
Disk Method, revolve about Y AXIS.

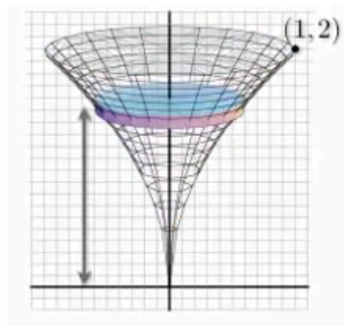
Consider $x = g(y) \geq 0$, continuous on [c,d], revolve about the y axis.





Example: The region bound by [The picture can't be displayed.], y=2 and y axis is revolved about the y axis. Find the volume:





Steps of Disk Method – Rather than *memorize* that disk method about x axis is dx and about the y axis is dy, it is going to help to think and really *understand* what you are doing. You would need to take slices of the solid perpendicular to the axis to of revolution get circular disks. This tells us what variable we will be integrating with respect to. Put the dx or dy in the integral formula first. In the 2D region, draw a representative rectangle perpendicular to the axis of revolution.

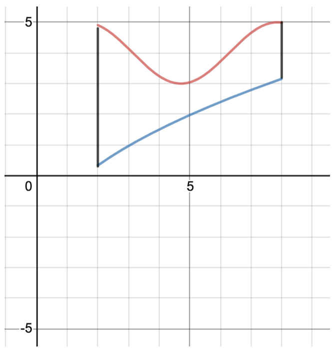
[The picture can't be displayed.] where thickness corresponds to dx or dy.

Washer Method

This is a version of the disk method that needs to be applied when the circular disk slices have a hole as shown in the animation Solid of revolution animation (washer) on the 5A page.

.

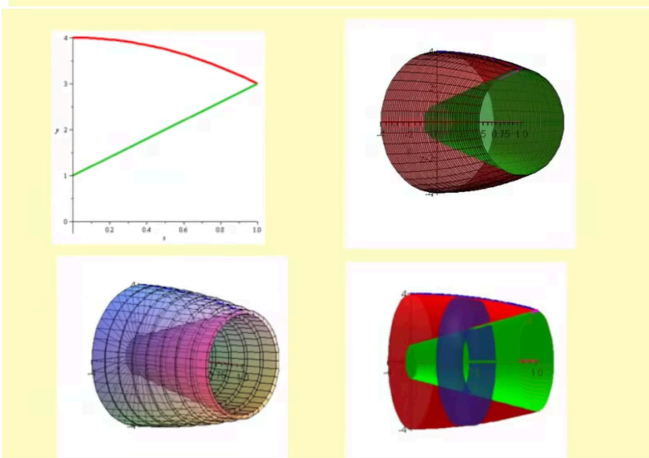
Derivation:



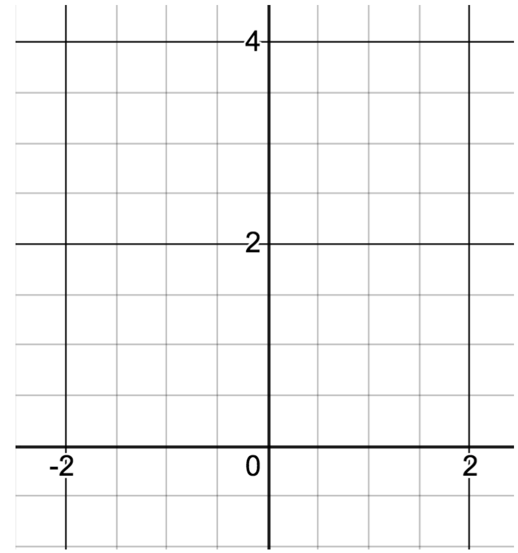
Example: (MathIsPower4U)

Determine the volume of the solid generated by the bounded region of given equation rotated about the x-axis.

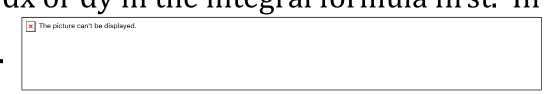
$y = -x^2 + 4, y = 2x + 1, x = 0$



<u>Example: Washers about the y axis</u>
Find the volume of the solid generated when the region bound by $y=4-x^2$ and $y=0$ for x [illegible] 1 is revolved about the y axis.



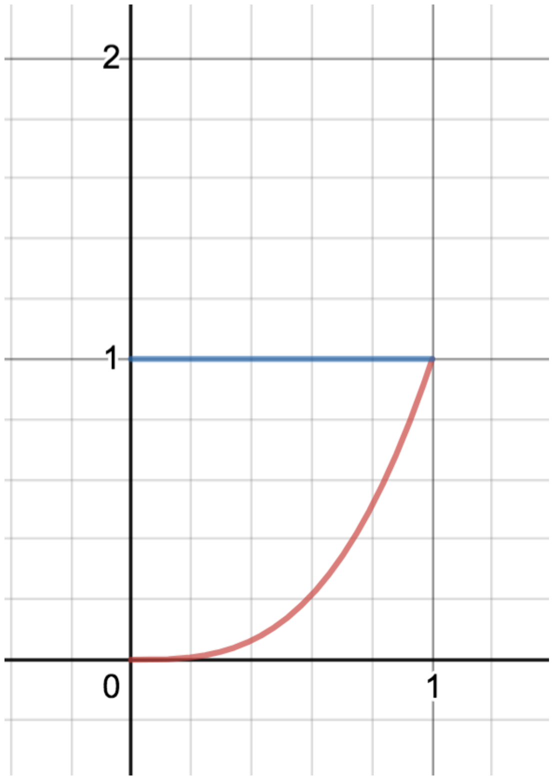
<u>Steps of Washer Method –</u> You would need to take slices of the solid perpendicular to the axis to of revolution get circular washers. This tells us what variable we will be integrating with respect to. Put the dx or dy in the integral formula first. In the 2D region, draw a representative rectangle perpendicular to the axis of revolution.



where thickness corresponds to dx or dy.

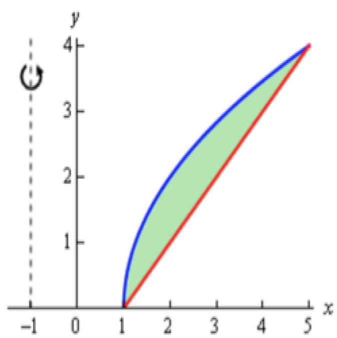
Example: Revolving about a line other than the x or y axis - Disks.

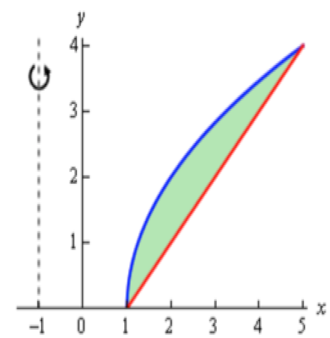
Find the volume of the solid generated when the region bound by $y = x^3$; $y = 1$ and the y axis is revolved about the line y=1

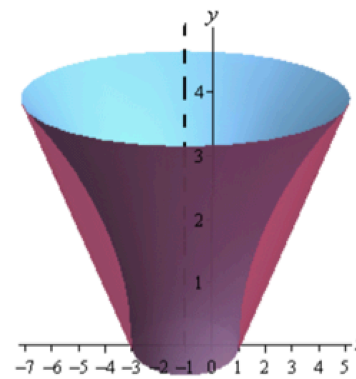


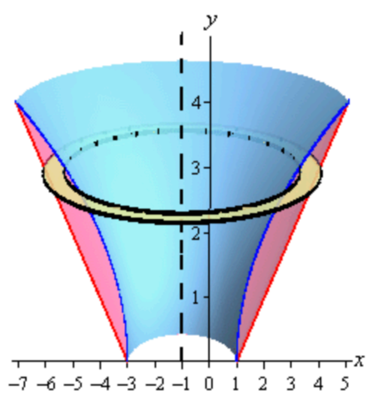
Example: Revolution about line other than axis - washers.

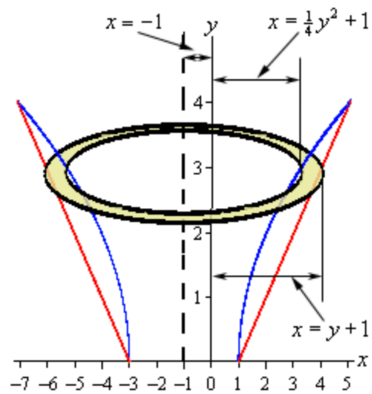
Find the volume of the solid generated when the region bound by The picture can't be displayed. is revolved about the line x=-1



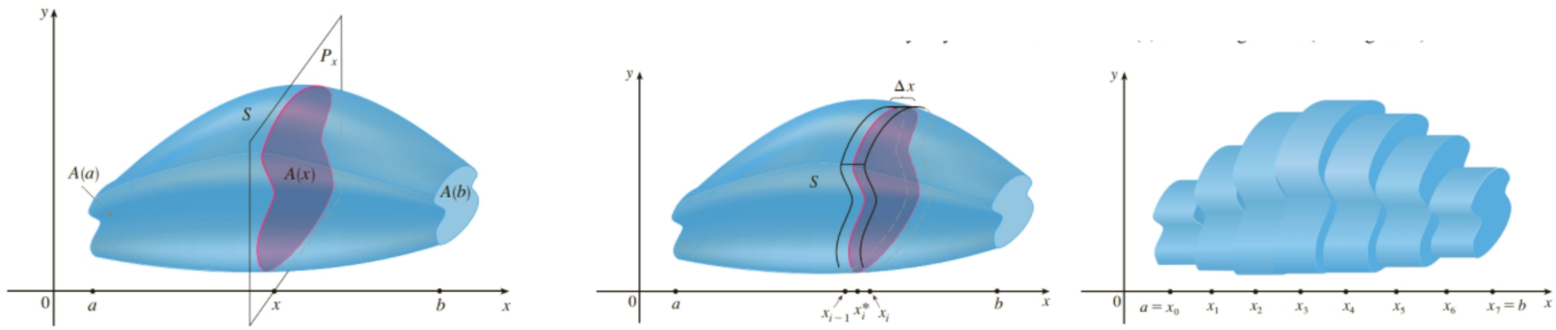






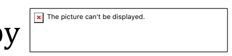


5.2 ii Volume by Slicing – Cross sections uniform shape.

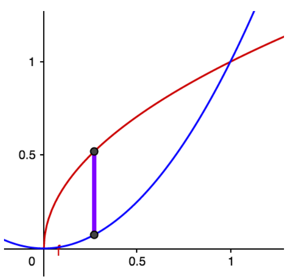


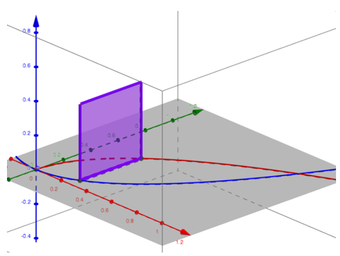
Definition of Volume Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of S in the plane P_x, through x and perpendicular to the x-axis, is $A(x)$, where A is a continuous function, then the **volume** of S is

$$V = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i^*)\,\Delta x = \int_a^b A(x)\,dx$$

<u>Example</u>: Find the volume of the solid whose base is the region enclosed by 

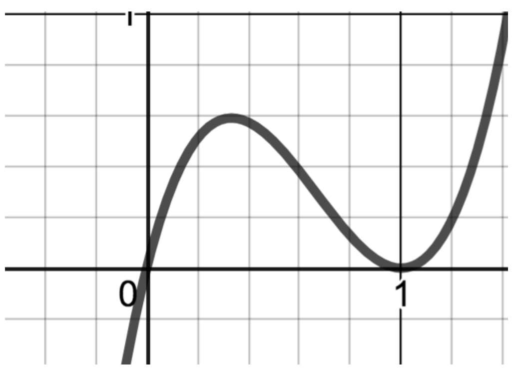
and having square cross sections perpendicular to the x axis.

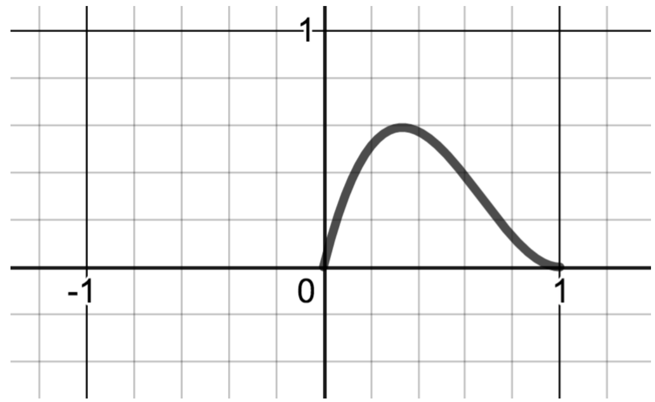




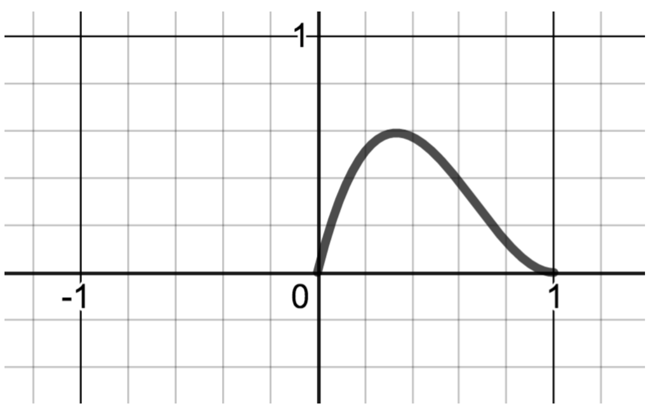
Geogebra: https://www.geogebra.org/m/XFgMaKTy Link in Canvas. Many other good examples and visualizations also.

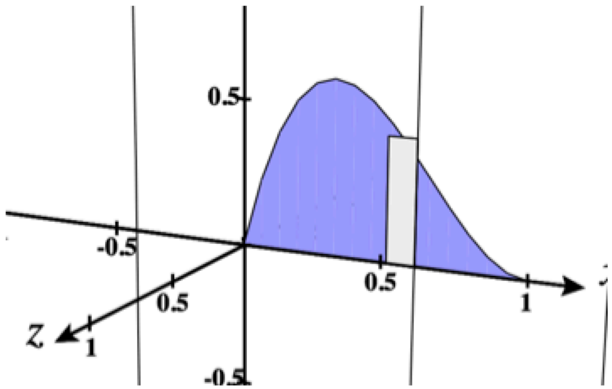
Find the volume of the solid formed by revolving the region bound by $f(x) = 4x^3 - 8x^2 + 4x$ and the x axis about the y axis.

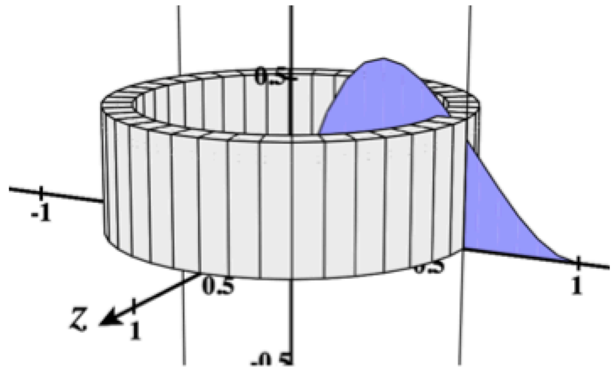


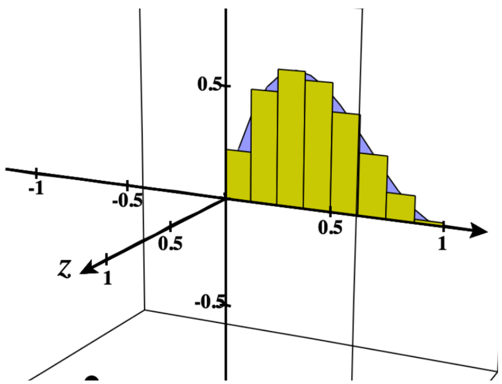


Is there another way?




0.5
-0.5
0.5
0.5
1
z
1
-0.5


0.5
-1
0.5
1
z
1
-0.5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-1
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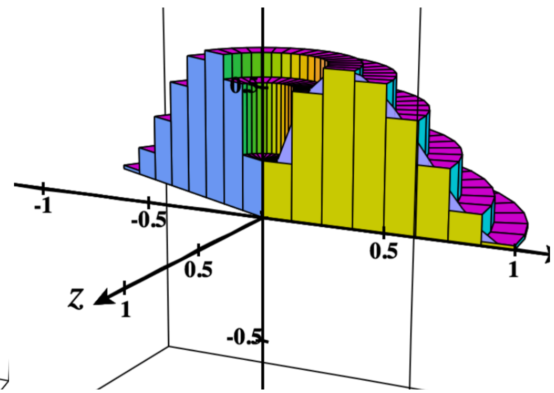
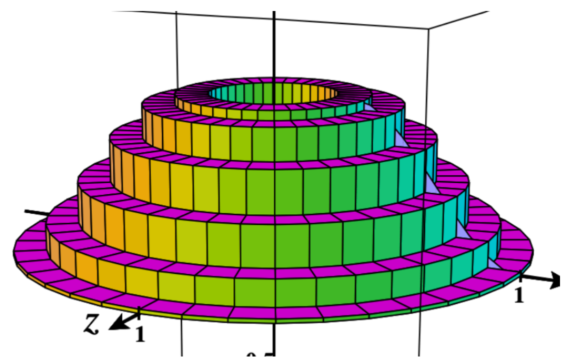
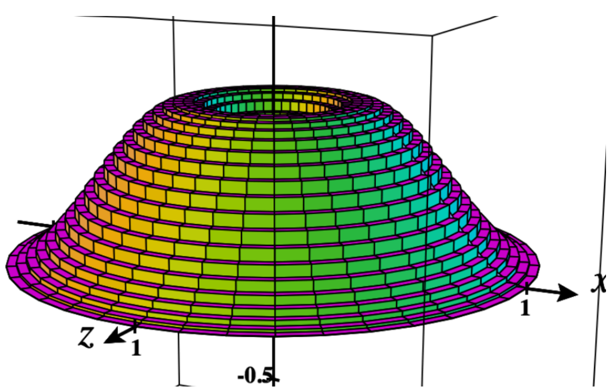
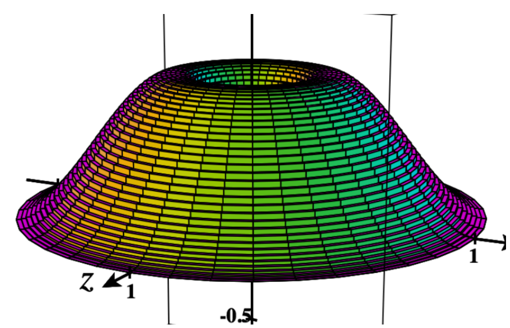
0.5
1
z
1
-0.5

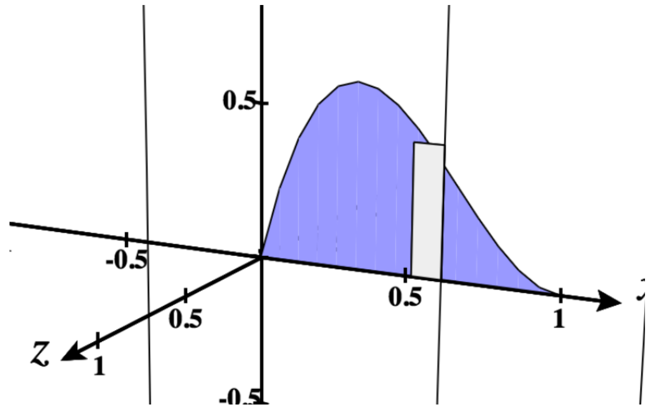

-1
-0.5
0.5
0.5
1
z
1
-0.5

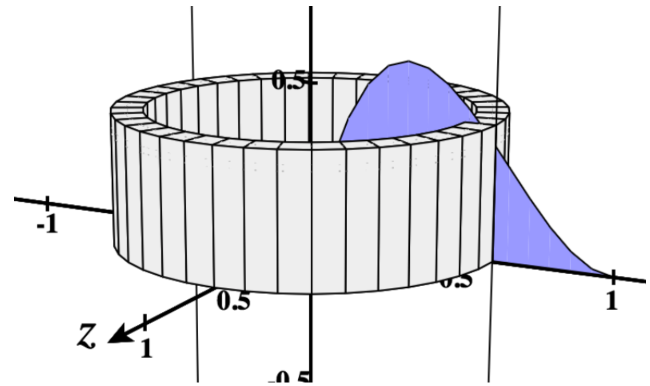

1
z
1


x
1
z
1
-0.5


1
z
1
-0.5

Derivation of the Method of Cylindrical Shells

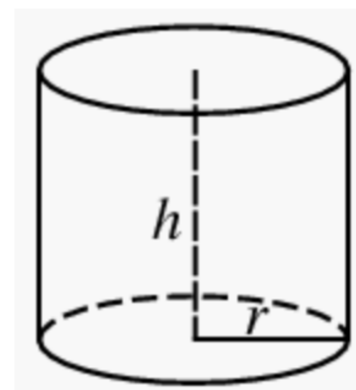




Volume of the i[th] shell ΔV_i: Can be derived many ways. In section 2.9, we used differentials:

35. (a) Use differentials to find a formula for the approximate volume of a thin cylindrical shell with height h, inner radius r, and thickness Δr.

The picture can't be displayed.



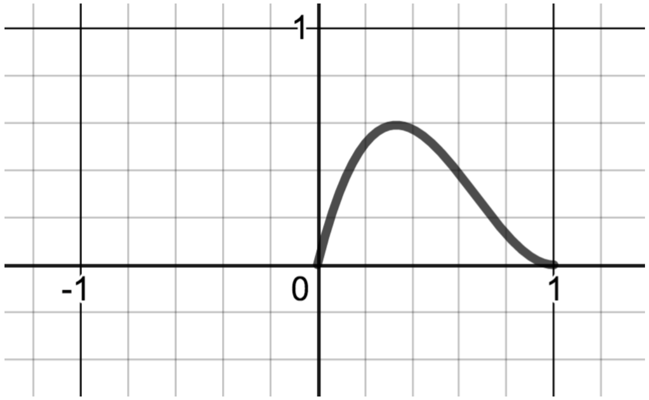
2 The volume of the solid in Figure 3, obtained by rotating about the y-axis the region under the curve $y = f(x)$ from a to b, is

$$V = \int_a^b 2\pi x f(x)\, dx \qquad \text{where } 0 \leqslant a < b$$

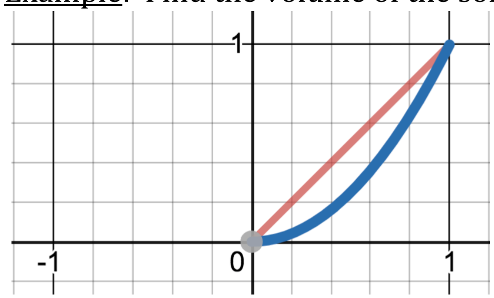
More generally:

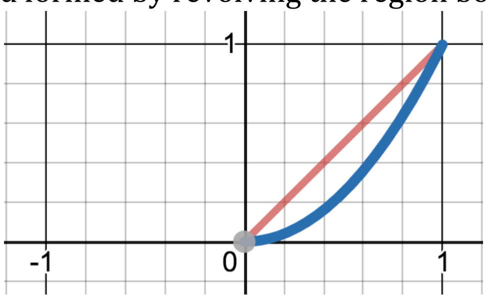
$$Volume = \int_a^b 2\pi(radius)(height)(thickness)$$

Back to example: $f(x) = 4x^3 - 8x^2 + 4x$ about y axis



Example: Find the volume of the solid formed by revolving the region bound by y=x and y=x² about the y axis.

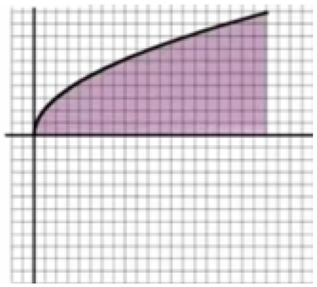




Shell Method Tips: Draw representative rectangle parallel to the axis of revolution.

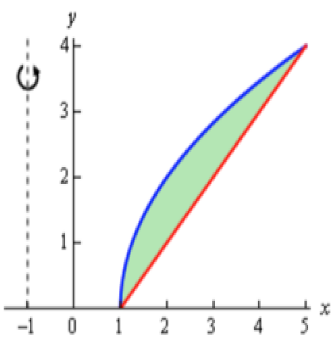
Example: Revolve about the x axis . (Done previously using disks)

The region bound by $f(x) = \sqrt{x}$, x=4 and x axis is revolved about the x axis. Find the volume:



Example: Revolution about line other than axis. (Done previously with washers)

Find the volume of the solid generated when the region bound by $y = 2\sqrt{x-1}$; $y = x-1$ is revolved about the line x=-1



Shell Method Tips: Draw representative rectangle parallel to the axis of revolution. The thickness determines dx or dy.

$$Volume = \int_a^b 2\pi(radius)(height)(thickness)$$

5.4 Work

Constant Force

When a body moves a distance d along a straight line as a result of being acted on by a force of constant magnitude F in the direction of motion, we define the work W done by the force on the body with the formula

Work = ____________ X _________________

In the SI system, units of force are kg-m/sec^2 or Newtons (N). The units of work are N-m or Joules.
In the British System, units of force are pounds(lbs). The units of work are ft-lbs.

Ex.

(b) How much work is done if a constant force of 50-lb is used to pull a cart 25 ft?_________________________________

(a) How much work is done lifting a 20 kg box 2 meters off the ground? _________________________________

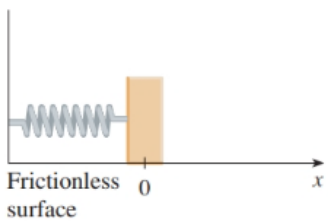
Variable Force

Suppose a particle moves along the x-axis from a to b acted upon by a continuous, variable force f(x).

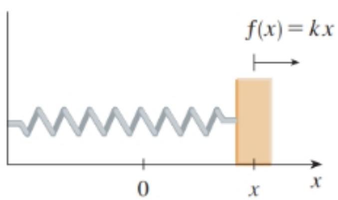
Ex. 2 When a particle is located a distance x feet from the origin, a force of $f(x) = x^2 + 2x$ pounds acts on it. How much work is done in moving it from x=1 to x=3?

HOOKE"S LAW for SPRINGS

The ____________________ required to maintain a spring stretched x units beyond its natural length is proportional to x.



(a) Natural position of spring



(b) Stretched position of spring

Example:

A spring has a natural length of 20 cm. A 40 N force is required to stretch (and hold the spring) to a length of 30 cm. How much work is done in stretching the spring from 35 cm to 38 cm??

Another Variable Force Example:
Ex: A 5-lb bucket is lifted from the ground into the air by pulling in 20 feet of rope at a constant speed. The rope weighs 0.08 lb/ft. How much work was spent lifting the bucket and rope?

Suppose that the bucket is leaking. It starts with 2 gallons (16 lb) of water in it and leaks at a constant rate. It finishes draining just as it reaches the top. How much work was spent lifting the water alone (neglect the rope and bucket.)